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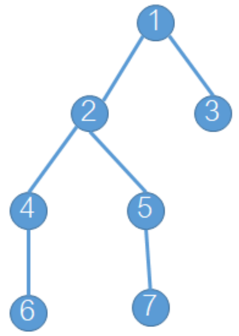
Please do not open this question set before you are allowed to do so.

4810-1185 Network Optimizations (Autumn 2018)

Midterm Problem 1

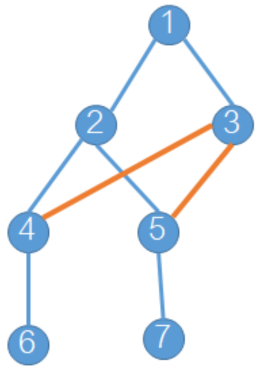
In this problem, we will try to use the tree decomposition technique to speeding up the calculation of the betweenness centrality.

Question 1.1: Calculate the betweenness centrality of all edges in the following social networks.



Question 1.2: Devise an algorithm that can calculate betweenness centrality of all edges in $O(||V||)$ when the input graph is a tree.

Question 1.3: Find a tree decomposition of the following social network. Your tree decomposition must have small treewidth.



Question 1.4: Calculate a betweenness centrality for all edges in the graph appeared in Question 1.3.

Question 1.5: Consider the following tree of bags in a tree decomposition. Discuss why all the paths from a node in Bag 1 to a node in Bag 3 must include some node in Bag 2.

Question 1.6: Give an idea for calculating the betweenness centrality on the graph with small treewidth. It would be more desirable if your algorithm's running time is in $O(f(k)||V||)$ when k is the treewidth and f is an arbitrary large function.

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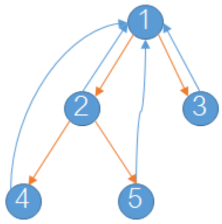
Midterm Problem 2

In this graph, we will calculate PageRank centrality for a web graph of a web portal service, such as Yahoo. In this graph, the users always begin browsing at the homepage. Then, they will go to the web portal of a particular topic (such as sports or IT), and, from the web portal of a particular topic, go to the web portal of a sub-topic (such as Football, Sumo, Blockchain, or AI). From all pages we can browse back to the homepage.

Our web graphs look like the following graph:



Question 2.1: Calculate PageRank for all webs in the above graph.

Question 2.2: Devise an algorithm to calculate PageRank centrality for such a graph in $O(||V||)$.

From next question, we will calculate the personalized PageRank of such web graphs. Suppose that $\beta = 0.5$.

Question 2.3: Calculate the matrix M for the web graph in Question 2.1.

Question 2.4: Calculate the matrix $(I - (1 - \beta)M)$ for the web graph in Question 2.1.

Question 2.5: Which elimination order is the most suitable for the matrix $(I - (1 - \beta)M)$?

Question 2.6: Give an algorithm for PageRank based on LU decomposition. Discuss why the decomposition will finish in $O(||V||)$.

Hint: The process of LU Decomposition is as follows:

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 9 \\ 1 & 1 & -1 & 1 \\ 3 & 11 & 5 & 35 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 1 & 9 \\ 0 & -2 & -2 & -8 \\ 0 & 2 & 2 & 8 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 1 & 9 \\ 0 & -2 & -2 & -8 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & -3 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

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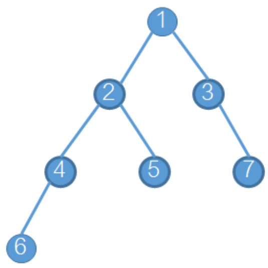
Midterm Problem 3

Consider the following social network:



Question 3.1: Based on the properties of the k-way clustering, guess the clustering result when the desired number of clustering is 2. (You do not need to really calculate k-way clustering, but you have to give a reason for your guess.)

Question 3.2: Based on your answer in Question 3.1, assume that the input graph is a tree. Design an algorithm that finishes in $O(||V||)$ for the 2-group cluster. Discuss why your result would be similar to that of k-way clustering.

Question 3.3: Based on your answer in Question 3.1, design an algorithm that finishes in $O\left(||V||^3\right)$ for the 3-group cluster. Discuss why your result would be similar to that of k-way clustering.

Question 3.4: Based on your answer in Question 3.1, design an algorithm that finishes in $O\left(||V||^{\ell+1}\right)$ for the ℓ-group cluster. Discuss why your result would be similar to that of k-way clustering.

Question 3.5: Discuss why your algorithm in Question 3.4 is not a fixed-parameter tractability algorithm, even when the parameter that we assume small is ℓ.